Control Lab  
Bode Plot &State Space

|  |  |  |
| --- | --- | --- |
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Part 1

## Introduction

We’re going to analyze a closed loop system analytically with hand analysis and experimentally using MATLAB

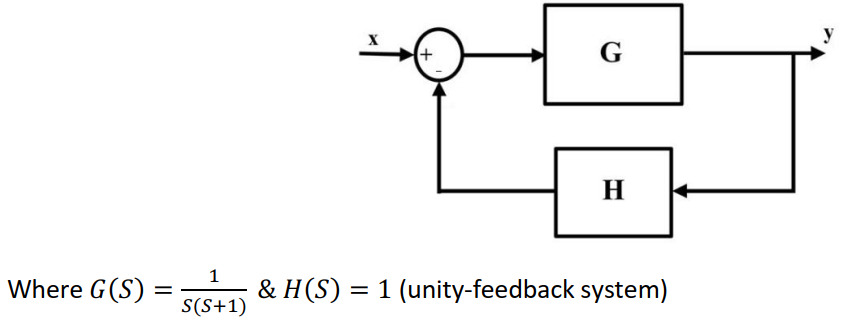


Figure 1 Closed loop System

As shown in figure 1, We’re going to investigate the time domain and frequency domain characteristics of a continuous time block placed in a feedback configuration.

## Q1)

### Code:

%--------Q1--------

% Define the open-loop transfer function G(s)

num\_G = 1;

den\_G = [1 1 0]; % s(s+1) = s^2 + s

G\_S = tf(num\_G, den\_G)

% Define the feedback transfer function H(s)

num\_H = [1];

den\_H = [1]; % Unity Feedback

H\_S = tf(num\_H, den\_H)

### Output:

## Q2)

### Theoretical:

To analyze the output of the block for a unit step input, we will perform a hand analysis.

### Step 1: Define the Input

The unit step input is defined as:

### Step 2: Calculate the Output

To find the output across the block , we can use the formula:

Substituting the definitions:

### Step 3: Perform Inverse Laplace Transform

### Step 4: Finding Inverse Laplace Transform

Now we can find the inverse Laplace transforms of each term separately:

### Interpretation

1. The term indicates a negative offset in the steady state before considering the effects of the ramp and exponential decay.
2. The term represents a linear rise in the output.
3. The term represents a decaying exponential that pulls the response down towards the steady-state value over time.

To analyze the stability of the system we’re going to:

### Step 1: Identify the Denominator

The transfer function is given by:

The denominator is:

This gives us two equations:

1. →

The system is **criticaly stable** as we have a pole at zero and a pole at -1 . then this process will

give a ramp response.

Therefore, the overall conclusion is that the transfer function exhibits marginal stability due to the pole on the imaginary axis, meaning while the system does not exhibit uncontrollable growth, it does not settle to a steady state.

### Code:

%--------Q2-------- Step Response of G(s) (Open-Loop)

% Plot step response of G(s)

draw\_step(G\_S, 'Open-Loop System G(s)');  
  
  
function draw\_step(sys, sys\_name)

% Create figure

figure;

% Plot step response

step(sys);

title(['Step Response of ', sys\_name]);

grid on;

% Display poles

poles = pole(sys);

disp(['Poles of ', sys\_name, ':']);

disp(poles);

% Check stability

if all(real(poles) < 0)

disp('System is stable (all poles in LHP)');

elseif any(real(poles) > 0)

disp('System is unstable (at least one pole in RHP)');

else

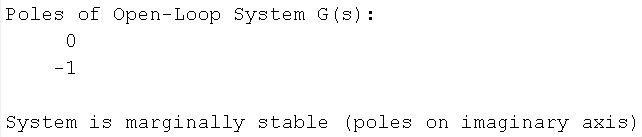
disp('System is marginally stable (poles on imaginary axis)');

end

end

### Output:

Figure 2 Gain Step Response



**As shown in figure 2, We can say this process is not stable (critical stable ) . And this expected as we have a pole in the (jw) axis** .

## Q3)

### Theoretical

Assume the open-loop transfer function is:

The closed-loop transfer function is derived using the following formula:

Thus, the closed-loop transfer function now becomes:

### Code:

%--------Q3-------- Closed-Loop Analysis

disp('Closed-Loop TF using feedback():');

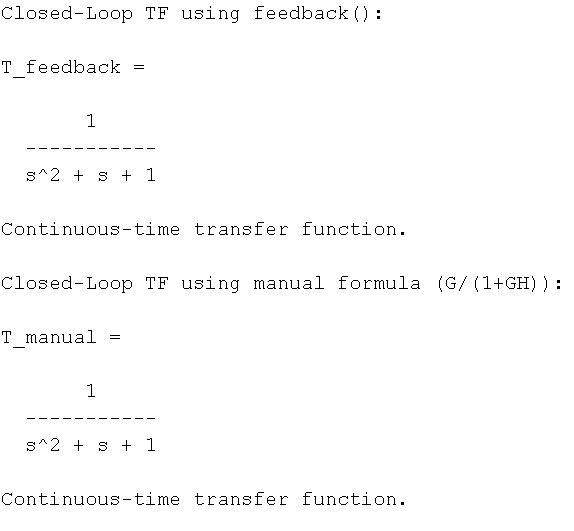
T\_feedback = feedback(G\_S, H\_S)

disp('Closed-Loop TF using manual formula (G/(1+GH)):');

T\_manual = (1 / (1 + G\_S \* H\_S)) \* G\_S; % Equivalent to T(s) = G/(1+GH)

T\_manual = minreal(T\_manual) % Cancel common terms

### Output:



**The output and the hand analysis is the same.**

## Q4)

### Theoretical

We derived the closed-loop transfer function:

To determine stability, we need to find the poles of the transfer function :

To solve for , we can use the quadratic formula:

where .

Substituting these values:

The poles are:

A system is considered stable as all poles in the RHP. It is a under damped system . And also we have some damped oscillations.

**Yes it is expected as the all poles in the RHP . and the poles has a imaginary part . and it is**

**Called a under damped system . zeta <1**

Time-Domain Specifications

* **Settling Time** :
* **Peak Time** :
* **Rise Time** :
* **Maximum Overshoot** :

### Code:

%--------Q4-------- Step Response of T(s) (Closed-Loop)

% Plot step response of T(s)

draw\_step(T\_feedback, 'Closed-Loop System T(s)');

### Output:

System: Closed-Loop System T(s)

Poles: -0.5-0.86603i -0.5+0.86603i

Stability: stable (all poles in LHP)

Over shoot MP: 16.2929% at t = 3.592 sec

Damping ratio (ζ): 0.500

Natural frequency (ωn): 1.000 rad/s

Settling time (2%): 8.1051 sec

Rise time (10-90%): 1.6579 sec

Steady-state value: 1.0014

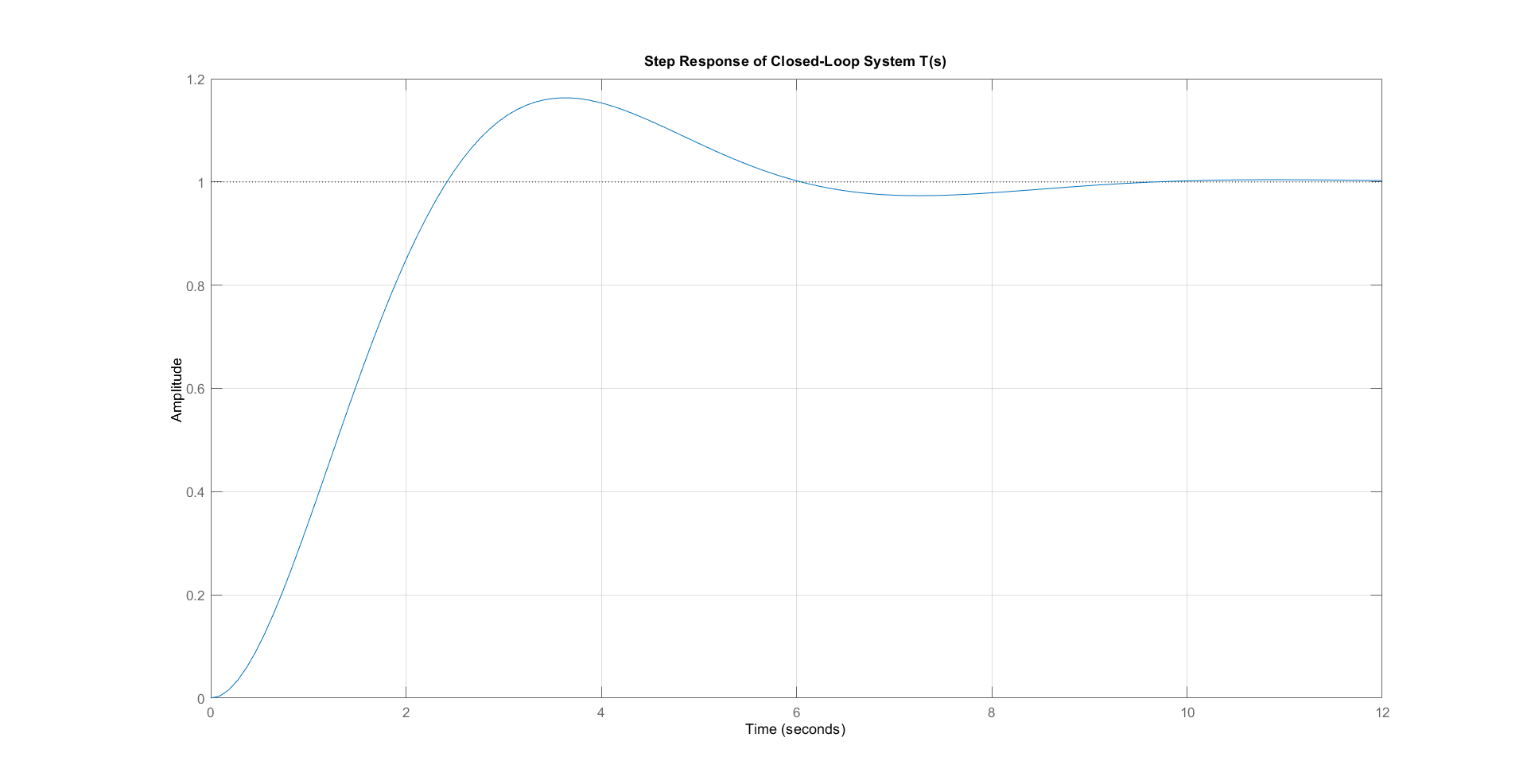


Figure 3 closed loop Step Response

**As shown in figure 3, the plot shows that it has some oscillations and the location of the pole in the experiment is equal to my hand analysis. And the damped oscillations is almost equal.**

### Theoretical vs Experimental:

|  |  |  |
| --- | --- | --- |
|  | Theoretical | Experimental |
| **Settling Time** | 8 sec | 8.1051 sec |
| **Peak Time** | 3.6275 sec | 3.592 sec |
| **Rise Time** | 2.4183 sec | 1.6579 sec |
| **Maximum Overshoot** | 16.3% | 16.2929% |
| **Ess** | 1 | 1.0014 |

**We can say the theoretical is almost equal to the Experimental.**

## Q5)

### Code:

function [poles] = draw\_poles(sys)

% Create figure

figure;

% Plot pole-zero map

pzmap(sys);

title(['Pole-Zero Map of: ' inputname(1)]);

grid on;

% Get poles

poles = pole(sys);

% Display poles

disp(['Poles of ' inputname(1) ':']);

disp(poles);

% Damping characteristics (for complex poles)

if ~isreal(poles)

[wn, zeta] = damp(sys);

fprintf('Damping ratio (?): %.3f\n', zeta(1));

fprintf('Natural frequency (?n): %.3f rad/s\n', wn(1));

end

end

### Output:

Poles of T\_feedback:

-0.5000 + 0.8660i

-0.5000 - 0.8660i

Damping ratio (ζ): 0.500

Natural frequency (ωn): 1.000 rad/s

### Theoretical vs Experimental:

From Q4:

|  |  |  |
| --- | --- | --- |
|  | Theoretical | Experimental |
| **Poles** |  | -0.5000 ± 0.8660i |
| **Natural frequency** | 1 rad/s | 1.000 rad/s |
| **Damping ratio** | 0.5 | 0.500 |

We can conclude that the experimental poles are ≈ theoretical ones so it agrees with the results from Q4.

## Q6)

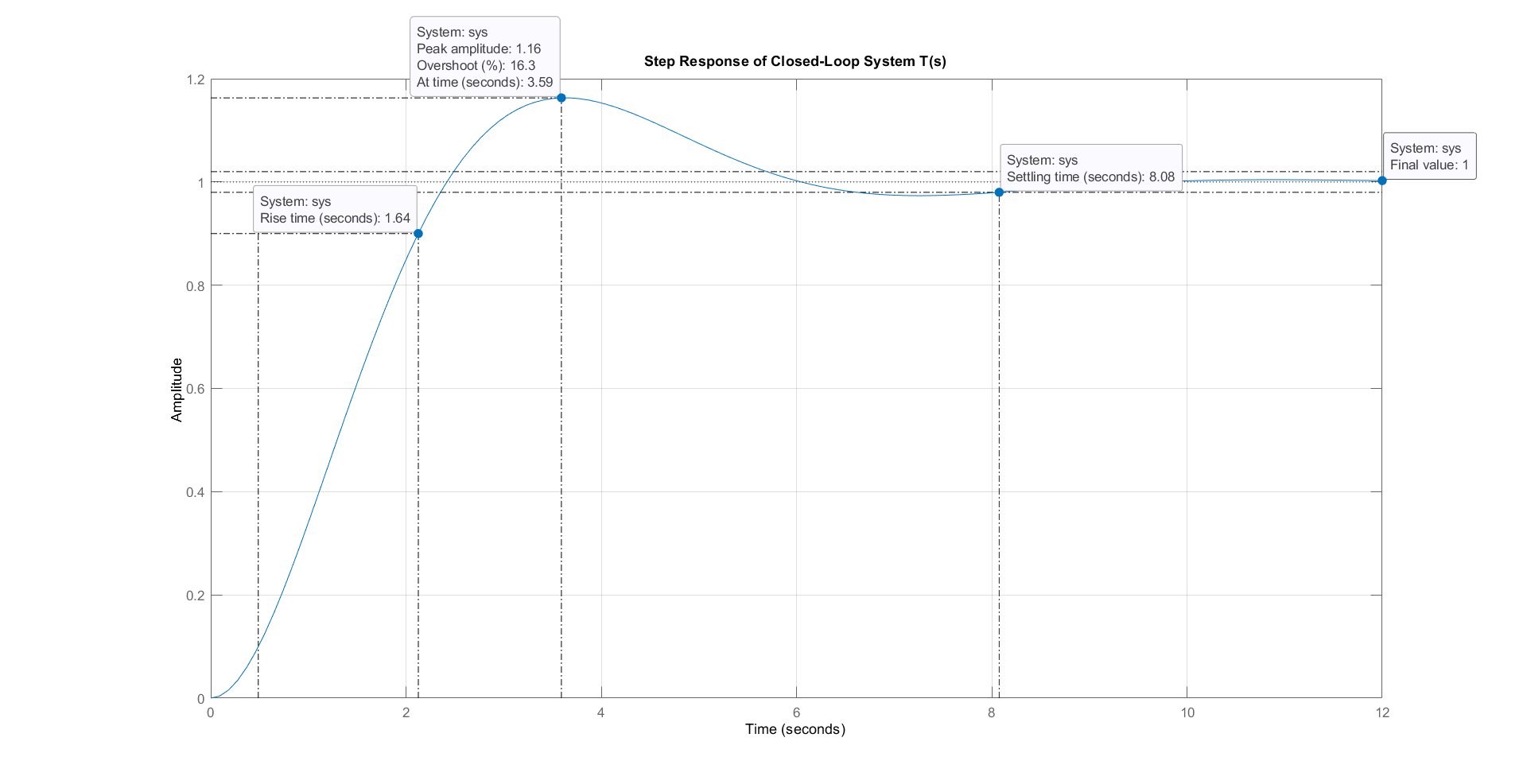


Figure 4 characteristics of the step response

From Q4)

|  |  |
| --- | --- |
| characteristic | Value |
| **Settling Time** | 8.1051 sec |
| **Peak Time** | 3.592 sec |
| **Rise Time** | 1.6579 sec |
| **Maximum Overshoot** | 16.2929% |
| **Ess** | 1.0014 |

This also answers Q7)

As the Ess is 1.0014, And yes it agrees with the second order system.

## Q8

### Code:

function [ess, t\_out, y\_out] = draw\_ramp(sys, t\_end, zoom\_time)

{

% Set defaults if not provided

if nargin < 2

t\_end = 100;

end

if nargin < 3

zoom\_time = 700;

end

% Create time vector

t = 0:0.1:t\_end;

%getting the ramp

ramp = tf(1,[1 0]);

% Get response data

[y\_sys, t\_sys] = step(sys.\*ramp, t);

[y\_ideal, t\_ideal] = step(ramp, t);

% Create figure with three subplots

figure;

% Subplot 1: Ideal ramp input

subplot(2,1,1);

plot(t\_ideal, y\_ideal, 'b');

hold on;

plot(t\_sys, y\_sys, 'r--');

title('Ramp Response');

xlabel('Time (sec)');

ylabel('Amplitude');

legend('Ideal', 'System', 'Location', 'northwest');

grid on;

hold off;

% Subplot 2: Zoomed comparison

subplot(2,1,2);

plot(t\_ideal, y\_ideal, 'b');

hold on;

plot(t\_sys, y\_sys, 'r--');

xlim([zoom\_time-50 zoom\_time+50]);

title(['Zoomed Comparison at t = ', num2str(zoom\_time), ' sec']);

xlabel('Time (sec)');

ylabel('Amplitude');

legend('Ideal', 'System', 'Location', 'northwest');

grid on;

hold off;

end

### Output:

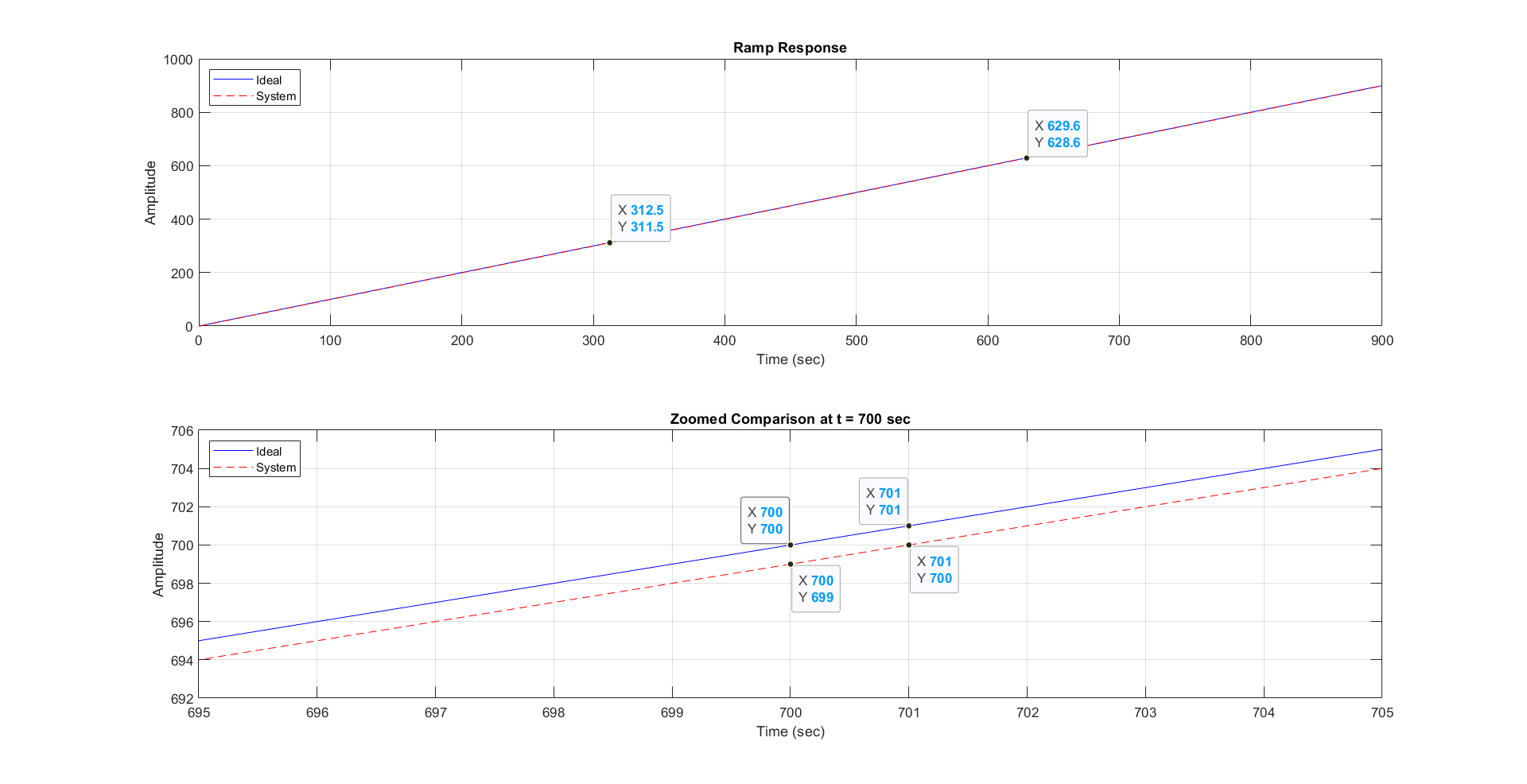


Figure 5 System Ramp Response

Steady-state error (ess): 1

### Hand analysis:

As shown in figure 5, the theoretical agrees with the experimental.

## Q9)

### Theo

### Code:

function [Gm, Pm, Wgc, Wpc] = draw\_Bode\_Plot(sys)

% BODE\_PLOT Analyzes system stability margins and compares margin()

% Bode\_Plot(sys)

%

% Input:

% sys - Transfer function (tf object or state-space model)

% Outputs:

% Gm - Gain margin (dB)

% Pm - Phase margin (degrees)

% Wgc - Gain crossover frequency (rad/sec)

% Wpc - Phase crossover frequency (rad/sec)

% Create margin plot

figure;

margin(sys);

grid on;

% Get stability margins

[Gm, Pm, Wgc, Wpc] = margin(sys);

% Display results

disp(['=== Stability Margins for ' inputname(1) ' ===']);

disp(['Gain Margin: ', num2str(Gm), ' dB at ', num2str(Wgc), ' rad/s']);

disp(['Phase Margin: ', num2str(Pm), '° at ', num2str(Wpc), ' rad/s']);

end

### Output:

Figure 6 Systme Bode Plot

=== Stability Margins for ===

Gain Margin: Inf dB at Inf rad/s

Phase Margin: 51.8273° at 0.78615 rad/s

**As shown in figure 6, We notices that the wps at infinity so the gain margin will be equal to infinity, but the phase margin we had small diffrence between the hand analysis and the experinital by almost (5 degree)**

Part2

## Introduction:

We’re going to investigate the properties of the system described by a state space representation analytically with hand analysis and experimentally using MATLAB.

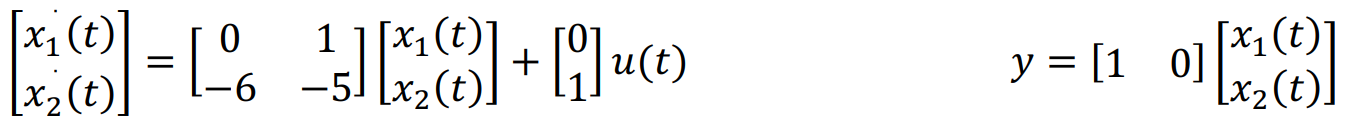


Figure 7 System State Space Representation

As shown in figure 7, We’re given the state space representation of the system.

## Q1)

### Theoritical

We have the following state-space equations:

**State Equation:**

Where:

**Output Equation:**

Where:

First, calculate :

Thus, we have:

### **Compute the Inverse**

The determinant of this matrix is:

Thus, the inverse is:

### **Multiply by**

Now we multiply by :

### **Compute**

Next, we find:

### **Final Transfer Function**

Therefore, the transfer function of the system is:

## Q2)

### Code:

% Given system matrices

A = [0 1; -6 -5];

B = [0; 1];

C = [1 0];

D = [0];

n = 2; % System order

sys = ss(A,B,C,D); % State Space model

x0 = [0; 1]; % Initial condition

% Q2: Transfer function conversion

[num, den] = ss2tf(A,B,C,D);

syms s

TF\_Manual = C\*inv(s\*eye(n)-A)\*B + D;

TF\_builtin = tf(num,den);

### Output:

TF\_Manual =

1/(s^2 + 5\*s + 6)

TF\_builtin =

1

-------------

s^2 + 5 s + 6

Continuous-time transfer function.

## Q3:

### Theoretical:

#### **First Entry:**

#### **Second Entry:**

#### **Third Entry:**

#### **Fourth Entry:**

Now, substitute back into :

### Inverse Laplace Transforms:

### Verify

When we evaluate :

it to yield directly.

### Code

% Q3: State transition matrix calculation

% Compute ?(s) = [sI - A]^-1

Phi\_s = inv(s\*eye(n) - A);

% Compute ?(t) by inverse Laplace transform

syms t

Phi\_t = ilaplace(Phi\_s);

% Verify ?(0) = I

Phi\_0 = subs(Phi\_t, t, 0);

% Display results

disp('State transition matrix in s-domain (?(s)):');

pretty(Phi\_s)

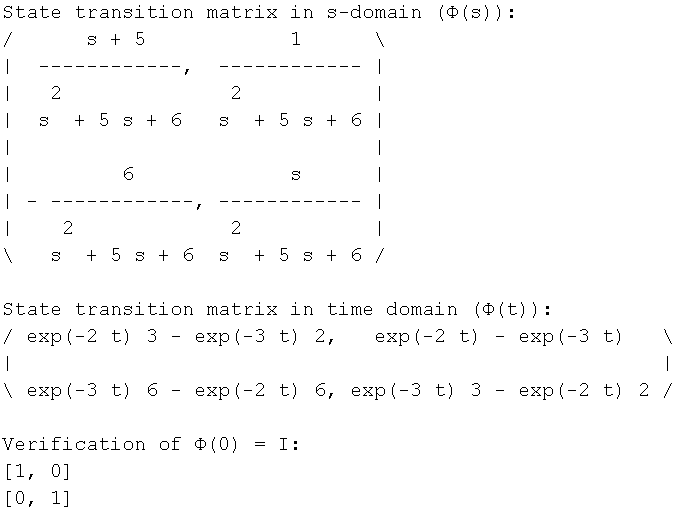
disp('State transition matrix in time domain (?(t)):');

pretty(Phi\_t)

disp('Verification of ?(0) = I:');

disp(Phi\_0);

### Output



it agrees with the theoretical analysis.

## Q4)

### Theoretical

Hence, we have:

Now let's calculate :

Using the previously defined :

Let's compute :

1. **First Row** of :
   * First Column:

### And so on

Now we can assemble :

Comparing and :

**we verify that .**

### Code:

% Q4: Verify that ??(t) = A?(t)

Phi\_dot = diff(Phi\_t, t); % Take time derivative of ?(t)

A\_Phi = A\*Phi\_t; % Multiply A with ?(t)

disp('Time derivative of state transition matrix (??(t)):');

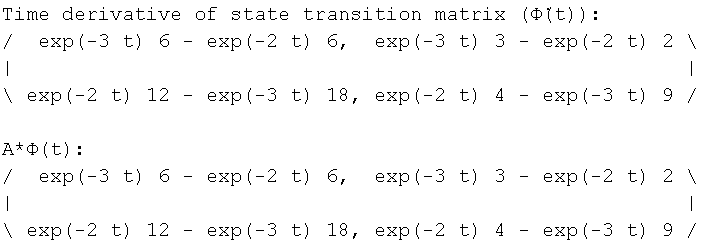
pretty(Phi\_dot)

disp('A\*?(t):');

pretty(A\_Phi)

disp('Verification successful: ??(t) = A?(t)');

### Output:



So we can see that it agrees with the theoretical analysis.

## Q5)

### Theoretical

### Step 1: Controllability

the controllability matrix :

1. First, compute :

So, we have:

### Rank of :

The determinant () is:

Since the determinant is non-zero, has full rank. Therefore, the system is controllable.

### Step 2: Observability

the observability matrix :

1. compute :

Thus, we have:

### Rank of :

The determinant of this matrix is non-zero. Thus, indicating that the rank is 2.

### Conclusion

**The system is controllable and observable**

### Code:

% Q5 Check Controllability and Observability

% Check Controllability

Co = ctrb(A, B); % Controllability matrix

rank\_Co = rank(Co);

disp('Controllability Matrix:');

disp(Co);

disp(['Rank of Controllability Matrix: ', num2str(rank\_Co)]);

if rank\_Co == n

disp('System is Controllable (as expected)');

else

disp('System is Not Controllable (unexpected for this system)');

end

% Check Observability

Ob = obsv(A, C); % Observability matrix

rank\_Ob = rank(Ob);

disp('Observability Matrix:');

disp(Ob);

disp(['Rank of Observability Matrix: ', num2str(rank\_Ob)]);

if rank\_Ob == n

disp('System is Observable (as expected)');

else

disp('System is Not Observable (unexpected for this system)');

end

### Output:

Controllability Matrix:

0 1

1 -5

Rank of Controllability Matrix: 2

System is Controllable (as expected)

Observability Matrix:

1 0

0 1

Rank of Observability Matrix: 2

System is Observable (as expected)

## Q6)

### Theoretical

The state equation is given by:

**Final Answers**

1. **Homogeneous state solution**:
2. **Output response**:

### Code:

% Q6: Unforced (Homogeneous) Response

disp('=== Unforced Response Analysis ===');

% Compute state solution x(t) = ?(t)\*x0

x\_t = Phi\_t \* x0;

disp('Unforced state solution x(t):');

pretty(x\_t)

% Compute output solution y(t) = C\*x(t) + D\*u(t)

% Since u(t)=0 for unforced response:

y\_t = C\*x\_t + D\*0;

disp('Unforced output response y(t):');

pretty(y\_t)

% Plot the results

t\_vals = linspace(0, 5, 500); % Time vector from 0 to 5 seconds

% Convert symbolic expressions to numeric functions

x1\_func = matlabFunction(x\_t(1));

x2\_func = matlabFunction(x\_t(2));

y\_func = matlabFunction(y\_t);

% Evaluate solutions

x1\_vals = arrayfun(x1\_func, t\_vals);

x2\_vals = arrayfun(x2\_func, t\_vals);

y\_vals = arrayfun(y\_func, t\_vals);

% Plot state responses

figure;

subplot(2,1,1);

plot(t\_vals, x1\_vals, 'b', 'LineWidth', 2);

hold on;

plot(t\_vals, x2\_vals, 'r--', 'LineWidth', 2);

title('Unforced State Response');

xlabel('Time (s)');

ylabel('State Values');

legend('x\_1(t)', 'x\_2(t)');

grid on;

% Plot output response

subplot(2,1,2);

plot(t\_vals, y\_vals, 'm', 'LineWidth', 2);

title('Unforced Output Response y(t)');

xlabel('Time (s)');

ylabel('Output y(t)');

grid on;

% Compare with MATLAB's built-in initial() function

[~,t\_num,x\_num] = initial(sys,x0,t\_vals(end));

y\_num = x\_num\*C'; % Equivalent to C\*x since D=0

% Display symbolic solutions

disp(' ');

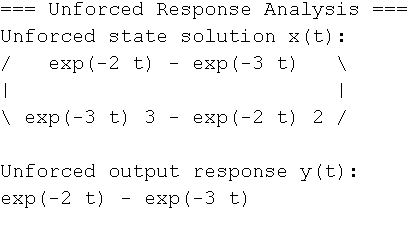
disp('Analytic Solutions:');

disp('x1(t) = '); pretty(x\_t(1))

disp('x2(t) = '); pretty(x\_t(2))

disp('y(t) = '); pretty(y\_t)

### Output:



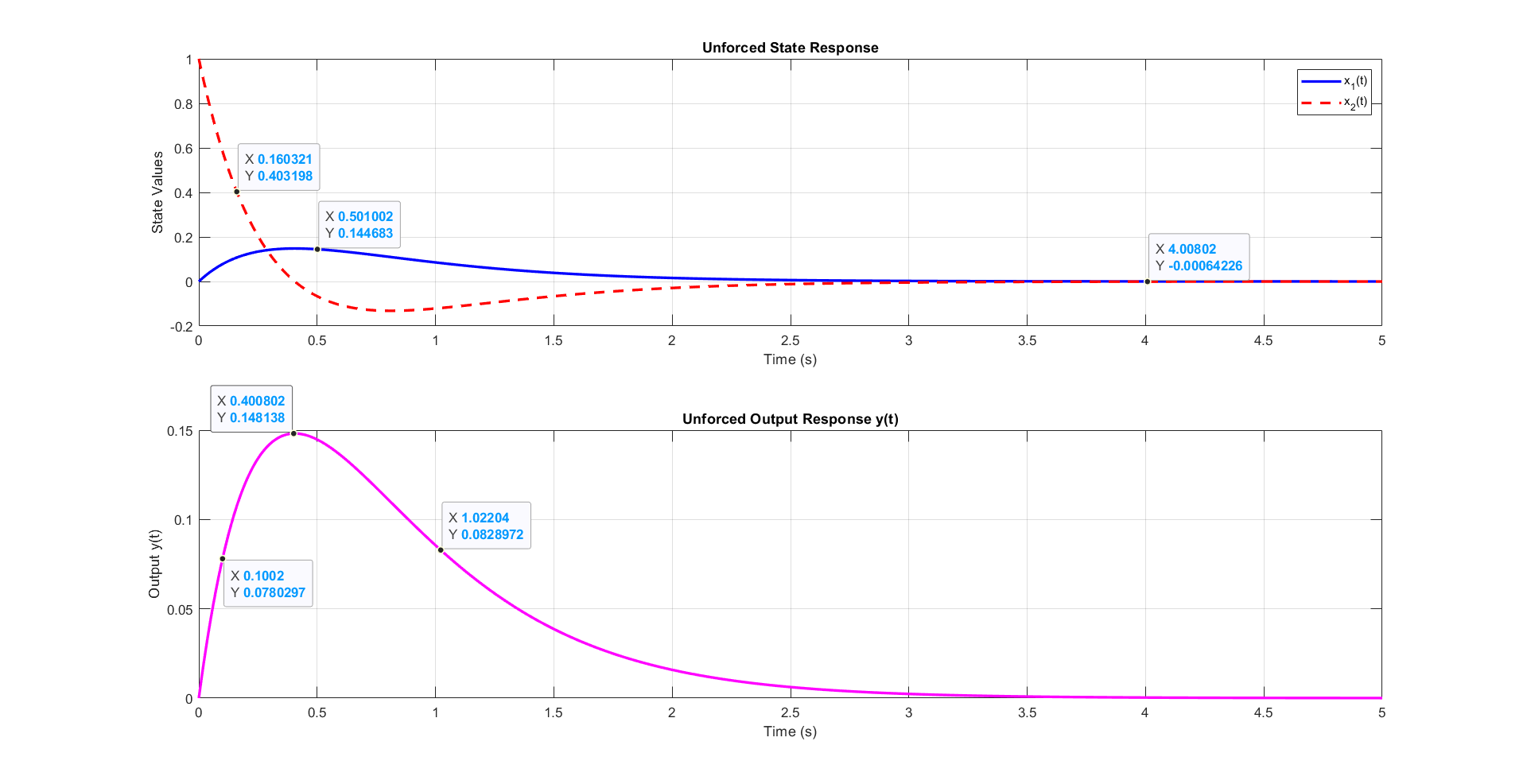


Figure 8 Unforced Output Plot

As shown in figure 8, The **experimental** unforced output agrees with the theoretical one.

## Q7)

### Code:

% Q7: Forced Response Analysis (Unit Step Input)

disp('=== Forced Response Analysis ===');

% Using Frequency Domain Approach

U\_s = 1/s; % Laplace transform of unit step

U\_t =ilaplace(U\_s);

% Compute forced component in frequency domain

X\_forced\_s = Phi\_s \* B \* U\_s;

% Convert to time domain

x\_forced\_t = ilaplace(X\_forced\_s);

% Total solution (homogeneous + forced)

x\_total\_t = x\_t + x\_forced\_t;

% Output solution

y\_total\_t = C\*x\_total\_t + D\*U\_t; % D\*u(t) where u(t)=1 for t>0

disp('Forced state solution (from step input):');

pretty(x\_forced\_t)

disp('Total state solution (unforced + forced):');

pretty(x\_total\_t)

% Direct evaluation using subs()

x1\_vals = double(subs(x\_total\_t(1), t, t\_vals));

x2\_vals = double(subs(x\_total\_t(2), t, t\_vals));

y\_vals = double(subs(y\_total\_t, t, t\_vals));

% Plot results

figure;

% State responses

subplot(2,1,1);

plot(t\_vals, x1\_vals, 'b', 'LineWidth', 2);

hold on;

plot(t\_vals, x2\_vals, 'r--', 'LineWidth', 2);

title('Total State Response (Step Input)');

xlabel('Time (s)');

ylabel('State Values');

legend('Analytic x\_1(t)', 'Analytic x\_2(t)');

grid on;

% Output response

subplot(2,1,2);

plot(t\_vals, y\_vals, 'm', 'LineWidth', 2);

hold on;

title('Total Output Response y(t) (Step Input)');

xlabel('Time (s)');

ylabel('Output y(t)');

legend('Analytic y(t)');

grid on;

disp(' ');

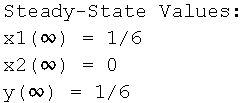
disp('Steady-State Values:');

disp(['x1(?) = ' char(ss\_x1)]);

disp(['x2(?) = ' char(ss\_x2)]);

disp(['y(?) = ' char(ss\_y)]);

### Output:



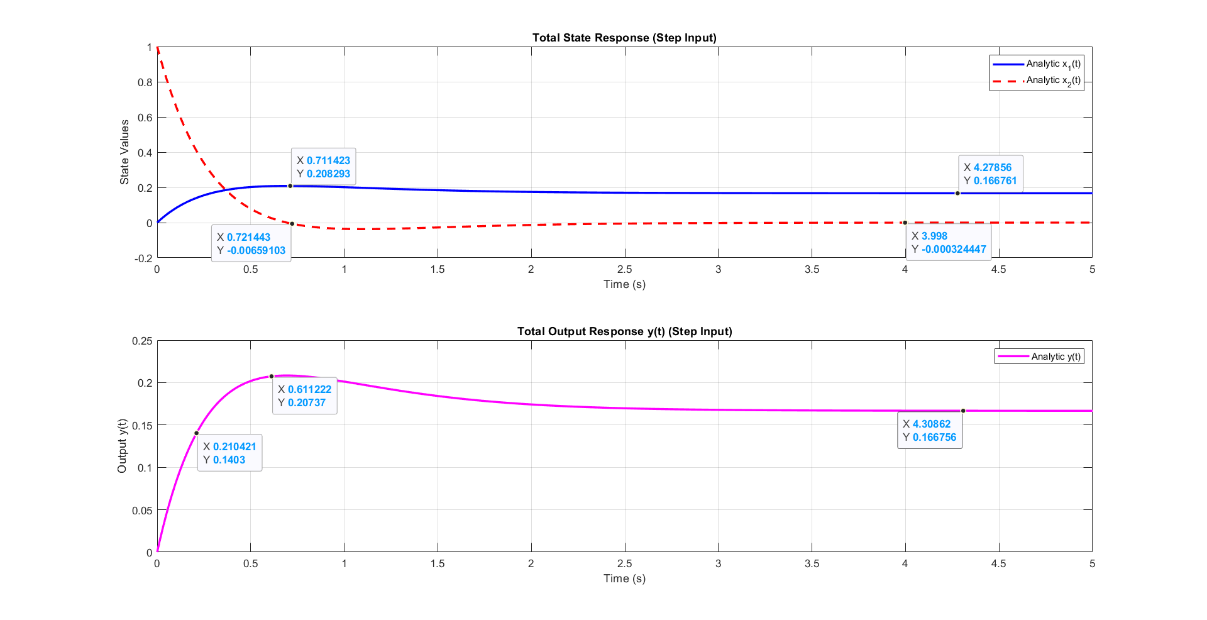
**.**

Figure 9 Forced Output Plot

As shown in figure 9,

**The result of the hand analysis is equal to the result of the experiment**

## Q8)

### Theoretical

Given:

Desired characteristic equation:

We want:

Let:

Compare with:

Matching coefficients:

Final result:

### Code:

% Q8: State Feedback Design

disp('=== State Feedback Design ===');

% Original system step response

figure;

step(TF\_builtin);

title('Original System Step Response');

grid on;

% Design specifications

zeta\_desired = 0.7; % Desired damping ratio

ts\_desired = 1; % Desired settling time (sec)

% Hand analysis to determine desired poles

wn = 4/(zeta\_desired\*ts\_desired); % Natural frequency from settling time

sigma = zeta\_desired\*wn; % Real part of poles

wd = wn\*sqrt(1-zeta\_desired^2); % Imaginary part

% Desired characteristic polynomial

desired\_poly = (s + sigma + 1i\*wd)\*(s + sigma - 1i\*wd);

desired\_poly = expand(desired\_poly);

% Convert to numerical polynomial

desired\_coeffs = sym2poly(desired\_poly);

% Hand calculation of K matrix

% Characteristic polynomial of A-BK: s^2 + (5+K2)s + (6+K1)

% Compare with desired polynomial: s^2 + 2\*zeta\*wn\*s + wn^2

K1 = desired\_coeffs(3) - 6; % From constant term

K2 = desired\_coeffs(2) - 5; % From s term

K = [K1 K2];

disp('Desired closed-loop poles:');

disp([-sigma+1i\*wd, -sigma-1i\*wd]);

disp('Feedback gain matarix K:');

disp(K);

% Verification

Ac = A - B\*K;

[num\_2, denum\_2] = ss2tf(Ac,B,C,D);

TF\_state\_feedback = tf(num\_2, denum\_2);

% Step response analysis

figure;

step\_info = stepinfo(TF\_state\_feedback);

step(TF\_state\_feedback);

title('System with State Feedback');

grid on;

disp('Closed-loop system performance:');

disp(['Settling Time: ', num2str(step\_info.SettlingTime), ' sec']);

disp(['Overshoot: ', num2str(step\_info.Overshoot), '%']);

### Output:

=== State Feedback Design ===

Desired closed-loop poles:

-4.0000 + 4.0808i -4.0000 - 4.0808i

Feedback gain matrix K:

26.6531 3.0000

Closed-loop system performance:

Settling Time: 1.0463 sec

Overshoot: 4.5986%

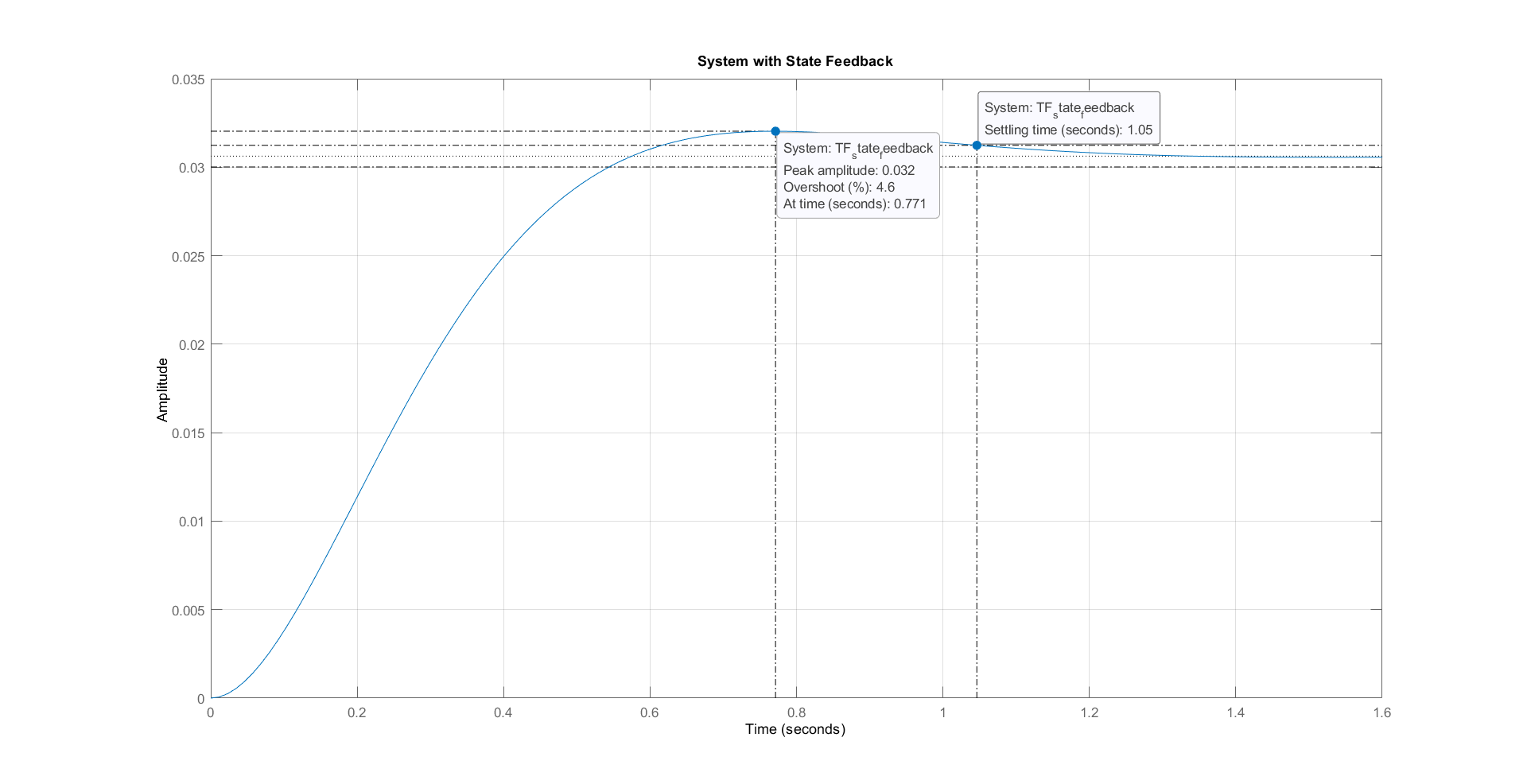


Figure 10 Desired System Response

As shown in figure 10,

|  |  |
| --- | --- |
| characteristic | Value |
| **Settling Time** | 1.05 sec |
| **Maximum Overshoot** | 4.6% |

Appendix

### Bode Plot Code:

clc;

clear;

close all;

%--------Q1-------- Define G(s) and H(s)

% Define the open-loop transfer function G(s)

num\_G = 1;

den\_G = [1 1 0]; % s(s+1) = s^2 + s

% Define the feedback transfer function H(s)

num\_H = [1];

den\_H = [1]; % Unity Feedback

[G\_S, H\_S] = create\_system(num\_G, den\_G, num\_H, den\_H)

%--------Q2-------- Step Response of G(s) (Open-Loop)

% Plot step response of G(s)

draw\_step(G\_S, 'Open-Loop System G(s)');

%--------Q3-------- Closed-Loop Analysis

disp('Closed-Loop TF using feedback():');

T\_feedback = feedback(G\_S, H\_S)

disp('Closed-Loop TF using manual formula (G/(1+GH)):');

T\_manual = (1 / (1 + G\_S \* H\_S)) \* G\_S; % Equivalent to T(s) = G/(1+GH)

T\_manual = minreal(T\_manual) % Cancel common terms

%--------Q4-------- Step Response of T(s) (Closed-Loop)

% Plot step response of T(s)

draw\_step(T\_feedback, 'Closed-Loop System T(s)');

%--------Q5-------- locations of the poles

draw\_poles(T\_feedback);

%--------Q8-------- Ramp Response

[ess, r\_t\_out, r\_y\_out] = draw\_ramp(T\_feedback, 700+200, 700);

%--------Q9-------- Frequency Response

[Gm, Pm, Wgc, Wpc] = draw\_Bode\_Plot(G\_S\*H\_S);

%-----------Functions------

function [G\_S, H\_S] = create\_system(num\_G, den\_G, num\_H, den\_H)

% CREATE\_SYSTEM Creates open-loop and feedback transfer functions

% [G\_S, H\_S] = create\_system(num\_G, den\_G, num\_H, den\_H)

%

% Inputs:

% num\_G - Numerator coefficients of G(s)

% den\_G - Denominator coefficients of G(s)

% num\_H - Numerator coefficients of H(s) (default: 1)

% den\_H - Denominator coefficients of H(s) (default: 1)

%

% Outputs:

% G\_S - Open-loop transfer function

% H\_S - Feedback transfer function

% Set default unity feedback if not specified

if nargin < 3

num\_H = 1;

den\_H = 1;

end

% Create transfer functions

G\_S = tf(num\_G, den\_G)

H\_S = tf(num\_H, den\_H)

end

function [wn, zeta,response\_info] = draw\_step(sys, sys\_name)

% DRAW\_STEP Plots step response and returns key performance metrics

% [response\_info] = draw\_step(sys, sys\_name)

%

% Inputs:

% sys - Transfer function (tf object)

% sys\_name - Name of the system for title (string)

%

% Outputs:

% response\_info - Structure containing:

% .poles - System poles

% .stability - Stability classification

% .peak\_response - Peak response value and time

% .settling\_time - Time to settle within 2% of final value

% .rise\_time - 10-90% rise time

% .steady\_state - Final steady-state value

% Figure with step response

% Create figure

figure;

% Get step response data

[y, t] = step(sys);

% Plot step response

step(sys);

title(['Step Response of ', sys\_name]);

grid on;

% Calculate response characteristics

response\_info = struct();

response\_info.poles = pole(sys);

% Stability determination

if all(real(response\_info.poles) < 0)

response\_info.stability = 'stable (all poles in LHP)';

elseif any(real(response\_info.poles) > 0)

response\_info.stability = 'unstable (at least one pole in RHP)';

else

response\_info.stability = 'marginally stable (poles on imaginary axis)';

end

% Peak response (overshoot)

[response\_info.peak\_response.value, peak\_idx] = max(y);

response\_info.peak\_response.time = t(peak\_idx);

% Steady-state value (last 10% of response)

steady\_state\_val = mean(y(end-round(length(y)\*0.1):end));

response\_info.steady\_state = steady\_state\_val;

% Settling time (within 2% of steady-state)

settled\_idx = find(abs(y - steady\_state\_val) > 0.02\*steady\_state\_val, 1, 'last');

if isempty(settled\_idx)

response\_info.settling\_time = 0;

else

response\_info.settling\_time = t(settled\_idx);

end

% Rise time (10% to 90% of steady-state)

rise\_start = find(y >= 0.1\*steady\_state\_val, 1);

rise\_end = find(y >= 0.9\*steady\_state\_val, 1);

if ~isempty(rise\_start) && ~isempty(rise\_end)

response\_info.rise\_time = t(rise\_end) - t(rise\_start);

else

response\_info.rise\_time = NaN;

end

% Display results in command window

disp(['System: ', sys\_name]);

disp(['Poles: ', num2str(response\_info.poles')]);

disp(['Stability: ', response\_info.stability]);

disp(['Over shoot MP: ', num2str(100\*(response\_info.peak\_response.value-1)), ...

'% at t = ', num2str(response\_info.peak\_response.time), ' sec']);

% Damping characteristics (for complex poles)

if ~isreal(response\_info.poles)

[wn, zeta] = damp(sys);

fprintf('Damping ratio (?): %.3f\n', zeta(1));

fprintf('Natural frequency (?n): %.3f rad/s\n', wn(1));

end

disp(['Settling time (2%): ', num2str(response\_info.settling\_time), ' sec']);

disp(['Rise time (10-90%): ', num2str(response\_info.rise\_time), ' sec']);

disp(['Steady-state value: ', num2str(response\_info.steady\_state)]);

end

function [poles] = draw\_poles(sys)

% DRAW\_POLES Plots pole-zero map and returns system poles

% [poles] = draw\_poles(sys)

%

% Input:

% sys - Transfer function (tf object) or state-space model

%

% Output:

% poles - Array of system poles

%

% Displays:

% - Pole-zero plot

% - Pole locations in command window

% - Stability information

% Create figure

figure;

% Plot pole-zero map

pzmap(sys);

title(['Pole-Zero Map of: ' inputname(1)]);

grid on;

% Get poles

poles = pole(sys);

% Display poles

disp(['Poles of ' inputname(1) ':']);

disp(poles);

% Damping characteristics (for complex poles)

if ~isreal(poles)

[wn, zeta] = damp(sys);

fprintf('Damping ratio (?): %.3f\n', zeta(1));

fprintf('Natural frequency (?n): %.3f rad/s\n', wn(1));

end

end

function [ess, t\_out, y\_out] = draw\_ramp(sys, t\_end, zoom\_time)

% DRAW\_RAMP Plots ramp response in three subplots

% [ess, t\_out, y\_out] = draw\_ramp(sys, t\_end, zoom\_time)

%

% Inputs:

% sys - Closed-loop transfer function (tf object)

% t\_end - End time for simulation (default: 100 sec)

% zoom\_time - Time to zoom in (default: 700 sec)

%

% Outputs:

% ess - Steady-state error

% t\_out - Time vector

% y\_out - System response vector

%

% Generates figure with three subplots:

% 1. Ideal ramp input

% 2. System response

% 3. Zoomed comparison at specified time

% Set defaults if not provided

if nargin < 2

t\_end = 100;

end

if nargin < 3

zoom\_time = 700;

end

% Create time vector

t = 0:0.1:t\_end;

%getting the ramp

ramp = tf(1,[1 0]);

% Get response data

[y\_sys, t\_sys] = step(sys.\*ramp, t);

[y\_ideal, t\_ideal] = step(ramp, t);

% Create figure with three subplots

figure;

% Subplot 1: Ideal ramp input

subplot(2,1,1);

plot(t\_ideal, y\_ideal, 'b');

hold on;

plot(t\_sys, y\_sys, 'r--');

title('Ramp Response');

xlabel('Time (sec)');

ylabel('Amplitude');

legend('Ideal', 'System', 'Location', 'northwest');

grid on;

hold off;

% Subplot 2: Zoomed comparison

subplot(2,1,2);

plot(t\_ideal, y\_ideal, 'b');

hold on;

plot(t\_sys, y\_sys, 'r--');

xlim([zoom\_time-50 zoom\_time+50]);

title(['Zoomed Comparison at t = ', num2str(zoom\_time), ' sec']);

xlabel('Time (sec)');

ylabel('Amplitude');

legend('Ideal', 'System', 'Location', 'northwest');

grid on;

hold off;

% Calculate steady-state error (use last 10% of simulation)

final\_idx = round(0.9\*length(t\_sys)):length(t\_sys);

ess = mean(y\_ideal(final\_idx) - y\_sys(final\_idx));

% Display results

disp(['Steady-state error (ess): ', num2str(ess)]);

% Return output data if requested

if nargout > 1

t\_out = t\_sys;

y\_out = y\_sys;

end

end

function [Gm, Pm, Wgc, Wpc] = draw\_Bode\_Plot(sys)

% BODE\_PLOT Analyzes system stability margins and compares margin()

% Bode\_Plot(sys)

%

% Input:

% sys - Transfer function (tf object or state-space model)

% Outputs:

% Gm - Gain margin (dB)

% Pm - Phase margin (degrees)

% Wgc - Gain crossover frequency (rad/sec)

% Wpc - Phase crossover frequency (rad/sec)

% Create margin plot

figure;

margin(sys);

grid on;

% Get stability margins

[Gm, Pm, Wgc, Wpc] = margin(sys);

% Display results

disp(['=== Stability Margins for ' inputname(1) ' ===']);

disp(['Gain Margin: ', num2str(Gm), ' dB at ', num2str(Wgc), ' rad/s']);

disp(['Phase Margin: ', num2str(Pm), '° at ', num2str(Wpc), ' rad/s']);

end

### State Space Code:

clc

clear all

close all

% Given system matrices

A = [0 1; -6 -5];

B = [0; 1];

C = [1 0];

D = [0];

n = 2; % System order

sys = ss(A,B,C,D); % State Space model

x0 = [0; 1]; % Initial condition

% Q2: Transfer function conversion

[num, den] = ss2tf(A,B,C,D);

syms s

TF\_Manual = C\*inv(s\*eye(n)-A)\*B + D

TF\_builtin = tf(num,den)

% Q3: State transition matrix calculation

% Compute ?(s) = [sI - A]^-1

Phi\_s = inv(s\*eye(n) - A);

% Compute ?(t) by inverse Laplace transform

syms t

Phi\_t = ilaplace(Phi\_s);

% Verify ?(0) = I

Phi\_0 = subs(Phi\_t, t, 0);

% Display results

disp('State transition matrix in s-domain (?(s)):');

pretty(Phi\_s)

disp('State transition matrix in time domain (?(t)):');

pretty(Phi\_t)

disp('Verification of ?(0) = I:');

disp(Phi\_0);

% Q4: Verify that ??(t) = A?(t)

Phi\_dot = diff(Phi\_t, t); % Take time derivative of ?(t)

A\_Phi = A\*Phi\_t; % Multiply A with ?(t)

disp('Time derivative of state transition matrix (??(t)):');

pretty(Phi\_dot)

disp('A\*?(t):');

pretty(A\_Phi)

disp('Verification successful: ??(t) = A?(t)');

% Q5 Check Controllability and Observability

% Check Controllability

Co = ctrb(A, B); % Controllability matrix

rank\_Co = rank(Co);

disp('Controllability Matrix:');

disp(Co);

disp(['Rank of Controllability Matrix: ', num2str(rank\_Co)]);

if rank\_Co == n

disp('System is Controllable (as expected)');

else

disp('System is Not Controllable (unexpected for this system)');

end

% Check Observability

Ob = obsv(A, C); % Observability matrix

rank\_Ob = rank(Ob);

disp('Observability Matrix:');

disp(Ob);

disp(['Rank of Observability Matrix: ', num2str(rank\_Ob)]);

if rank\_Ob == n

disp('System is Observable (as expected)');

else

disp('System is Not Observable (unexpected for this system)');

end

% Q6: Unforced (Homogeneous) Response

disp('=== Unforced Response Analysis ===');

% Compute state solution x(t) = ?(t)\*x0

x\_t = Phi\_t \* x0;

disp('Unforced state solution x(t):');

pretty(x\_t)

% Compute output solution y(t) = C\*x(t) + D\*u(t)

% Since u(t)=0 for unforced response:

y\_t = C\*x\_t + D\*0;

disp('Unforced output response y(t):');

pretty(y\_t)

% Plot the results

t\_vals = linspace(0, 5, 500); % Time vector from 0 to 5 seconds

% Convert symbolic expressions to numeric functions

x1\_func = matlabFunction(x\_t(1));

x2\_func = matlabFunction(x\_t(2));

y\_func = matlabFunction(y\_t);

% Evaluate solutions

x1\_vals = arrayfun(x1\_func, t\_vals);

x2\_vals = arrayfun(x2\_func, t\_vals);

y\_vals = arrayfun(y\_func, t\_vals);

% Plot state responses

figure;

subplot(2,1,1);

plot(t\_vals, x1\_vals, 'b', 'LineWidth', 2);

hold on;

plot(t\_vals, x2\_vals, 'r--', 'LineWidth', 2);

title('Unforced State Response');

xlabel('Time (s)');

ylabel('State Values');

legend('x\_1(t)', 'x\_2(t)');

grid on;

% Plot output response

subplot(2,1,2);

plot(t\_vals, y\_vals, 'm', 'LineWidth', 2);

title('Unforced Output Response y(t)');

xlabel('Time (s)');

ylabel('Output y(t)');

grid on;

% Compare with MATLAB's built-in initial() function

[~,t\_num,x\_num] = initial(sys,x0,t\_vals(end));

y\_num = x\_num\*C'; % Equivalent to C\*x since D=0

% Display symbolic solutions

disp(' ');

disp('Analytic Solutions:');

disp('x1(t) = '); pretty(x\_t(1))

disp('x2(t) = '); pretty(x\_t(2))

disp('y(t) = '); pretty(y\_t)

% Q7: Forced Response Analysis (Unit Step Input)

disp('=== Forced Response Analysis ===');

% Using Frequency Domain Approach

U\_s = 1/s; % Laplace transform of unit step

U\_t =ilaplace(U\_s);

% Compute forced component in frequency domain

X\_forced\_s = Phi\_s \* B \* U\_s;

% Convert to time domain

x\_forced\_t = ilaplace(X\_forced\_s);

% Total solution (homogeneous + forced)

x\_total\_t = x\_t + x\_forced\_t;

% Output solution

y\_total\_t = C\*x\_total\_t + D\*U\_t; % D\*u(t) where u(t)=1 for t>0

disp('Forced state solution (from step input):');

pretty(x\_forced\_t)

disp('Total state solution (unforced + forced):');

pretty(x\_total\_t)

% Direct evaluation using subs()

x1\_vals = double(subs(x\_total\_t(1), t, t\_vals));

x2\_vals = double(subs(x\_total\_t(2), t, t\_vals));

y\_vals = double(subs(y\_total\_t, t, t\_vals));

% Plot results

figure;

% State responses

subplot(2,1,1);

plot(t\_vals, x1\_vals, 'b', 'LineWidth', 2);

hold on;

plot(t\_vals, x2\_vals, 'r--', 'LineWidth', 2);

title('Total State Response (Step Input)');

xlabel('Time (s)');

ylabel('State Values');

legend('Analytic x\_1(t)', 'Analytic x\_2(t)');

grid on;

% Output response

subplot(2,1,2);

plot(t\_vals, y\_vals, 'm', 'LineWidth', 2);

hold on;

title('Total Output Response y(t) (Step Input)');

xlabel('Time (s)');

ylabel('Output y(t)');

legend('Analytic y(t)');

grid on;

% Display final steady-state values

ss\_x1 = limit(x\_total\_t(1), t, inf);

ss\_x2 = limit(x\_total\_t(2), t, inf);

ss\_y = limit(y\_total\_t, t, inf);

disp(' ');

disp('Steady-State Values:');

disp(['x1(?) = ' char(ss\_x1)]);

disp(['x2(?) = ' char(ss\_x2)]);

disp(['y(?) = ' char(ss\_y)]);

% Q8: State Feedback Design

disp('=== State Feedback Design ===');

% Original system step response

figure;

step(TF\_builtin);

title('Original System Step Response');

grid on;

% Design specifications

zeta\_desired = 0.7; % Desired damping ratio

ts\_desired = 1; % Desired settling time (sec)

% Hand analysis to determine desired poles

wn = 4/(zeta\_desired\*ts\_desired); % Natural frequency from settling time

sigma = zeta\_desired\*wn; % Real part of poles

wd = wn\*sqrt(1-zeta\_desired^2); % Imaginary part

% Desired characteristic polynomial

desired\_poly = (s + sigma + 1i\*wd)\*(s + sigma - 1i\*wd);

desired\_poly = expand(desired\_poly);

% Convert to numerical polynomial

desired\_coeffs = sym2poly(desired\_poly);

% Hand calculation of K matrix

% Characteristic polynomial of A-BK: s^2 + (5+K2)s + (6+K1)

% Compare with desired polynomial: s^2 + 2\*zeta\*wn\*s + wn^2

K1 = desired\_coeffs(3) - 6; % From constant term

K2 = desired\_coeffs(2) - 5; % From s term

K = [K1 K2];

disp('Desired closed-loop poles:');

disp([-sigma+1i\*wd, -sigma-1i\*wd]);

disp('Feedback gain matrix K:');

disp(K);

% Verification

Ac = A - B\*K;

[num\_2, denum\_2] = ss2tf(Ac,B,C,D);

TF\_state\_feedback = tf(num\_2, denum\_2);

% Step response analysis

figure;

step\_info = stepinfo(TF\_state\_feedback);

step(TF\_state\_feedback);

title('System with State Feedback');

grid on;

disp('Closed-loop system performance:');

disp(['Settling Time: ', num2str(step\_info.SettlingTime), ' sec']);

disp(['Overshoot: ', num2str(step\_info.Overshoot), '%']);